

# Robust Stereo Matching Based on Probabilistic Laplacian Propagation with Weighted Mutual Information

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## ABSTRACT

Conventional stereo matching methods provide the unsatisfactory results for stereo pairs under uncontrolled environments such as illumination distortions and camera device changes. A majority of efforts to address this problem has devoted to develop robust cost function. However, the stereo matching results by cost function cannot be liberated from a false correspondence when radiometric distortions exist. This paper presents a robust stereo matching approach based on probabilistic Laplacian propagation. In the proposed method, reliable ground control points are selected using weighted mutual information and reliability check. The ground control points are then propagated with probabilistic Laplacian. Since only reliable matching is propagated with the reliability of GCP, the proposed approach is robust to a false initial matching. Experimental results demonstrate the effectiveness of the proposed method in stereo matching for image pairs taken under illumination and exposure distortions.

**Keywords:** Laplacian propagation, mutual information, robust stereo matching, illumination, exposure

## 1. INTRODUCTION

Stereo matching algorithm aims to generate three-dimensional information by finding the correspondence of an image pair taken at different viewpoints of the same scene. However, conventional stereo matching algorithms might falls in a false matching problem in uncontrolled environments with a radiometric variation, illumination variation, and camera device variation.

To address this problem, a number of algorithms have been proposed focusing on a robust cost function. Normalized cross-correlation (NCC) is a window-based cost function compensating gain changes induced by affine-transform in color space [1]. However, NCC suffers from errors in object boundary causing fattening effect similar to other window-based costs. To alleviate this problem, Heo et al. [2] proposed an adaptive normalized cross correlation (ANCC) based on weight distribution around matching pixels. Recently, Kim et al. [3] proposed a Mahalanobis distance cross-correlation (MDCC) which uses a local color distribution for illumination invariant stereo matching. Mutual information (MI) has been considered a robust cost function handling a nonlinear relationship using joint entropy between stereo images [4, 5, 6]. Local order of intensities based cost functions, e.g., rank and census, also provide the robustness for the local variations [7, 8, 9]. Such a robust cost function remarkably improves the stereo matching performance under radiometric distortions. Nevertheless, no cost function can perfectly match for every pixels, especially in occlusion or homogeneous regions.

Motivated by this fact, propagation based stereo matching algorithms have been actively studied [10, 11, 12, 13, 14]. In the scheme, an unambiguous sparse (or semi-dense) disparity map is computed and then propagated into neighbors based on the premise that neighboring pixels with similar color have similar disparities. Laplacian propagation [15] is popularly used to propagate the unambiguous pixels, called as ground control point (GCP), into neighboring pixels. However, the conventional Laplacian propagation is very sensitive to GCPs since it assumes GCPs to be ground-truth. If selected GCPs are erroneous especially around an object boundary, the errors are propagated and consequently degrade final dense disparity map. In addition, when no CGPs exist around discontinuities, it may lead to the lack of information needed for appropriately guiding the subsequent propagation process [16].

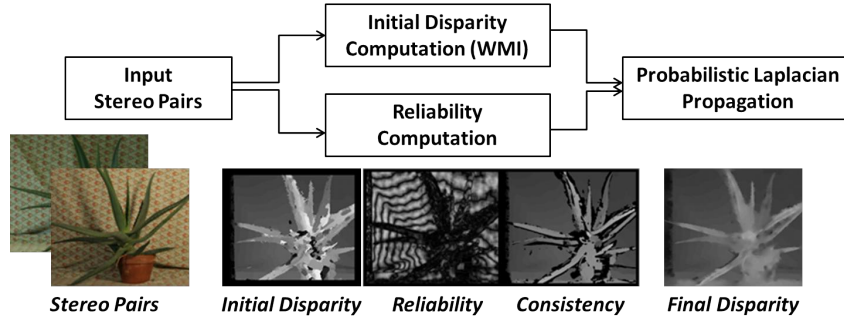


Figure 1. A framework of the proposed method.

To address these problems, this paper proposes probabilistic Laplacian propagation based stereo matching algorithm. In the proposed scheme, reliable GCPs are obtained using weighted mutual information (WMI) and reliability check process. The obtained GCPs are then propagated with probabilistic Laplacian model. This paper is organized as follows. Section 2 describes the proposed algorithm in detail. Experimental results are presented in Section 3. Lastly, Section 4 concludes this paper with discussion about future works.

## 2. PROBABILISTIC LAPLACIAN PROPAGATION WITH RELIABLE GROUND CONTROL POINTS

The framework of the proposed algorithm is shown in Fig. 1. First, initial disparity map is computed for both left and right images with WMI cost function. From the initial disparities, reliable GCPs are computed through reliability check process. The selected ground control points are propagated using probabilistic Laplacian propagation. Detailed descriptions of each step are presented in the following Sections.

### 2.1 Ground Control Point Computation

Reliable ground control points are computed by the combination of initial disparity estimation and reliability check. That is, only pixels of which disparity estimated is reliable are selected as ground control points. To estimate an initial disparity, any robust cost function can be used, e.g., ANCC [3], MI [4], Census [7]. However, these cost function possibly provides false disparity around an object boundary, and consequently leads severe error propagation problem [16].

To address this problem, we propose the WMI as a cost function. Let  $\mathbf{X}$  be a target patch,  $\hat{\mathbf{X}}$  be a reference patch, and  $f_p$  be a disparity at pixel  $p$ . Then, the cost function of the WMI  $C_w(p, p + f_p)$  is defined as follows:

$$\begin{aligned}
 C_w(p, p + f_p) &\triangleq H^w(\mathbf{X}_p) + H^w(\hat{\mathbf{X}}_{p+f_p}) - H^w(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p}) \\
 &\triangleq E_{\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p}} [\log(\frac{P(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p})}{P(\mathbf{X}_p)P(\hat{\mathbf{X}}_{p+f_p})})],
 \end{aligned} \tag{1}$$

where  $H^w(\mathbf{X}_p)$  and  $H^w(\hat{\mathbf{X}}_{p+f_p})$  are the entropies of window  $\mathbf{X}_p$  and window  $\hat{\mathbf{X}}_{p+f_p}$ , and  $H^w(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p})$  is the joint entropy of  $\mathbf{X}_p$  and  $\hat{\mathbf{X}}_{p+f_p}$ .  $\mathbf{X}_p$  and  $\hat{\mathbf{X}}_{p+f_p}$  mean that they are centered at position  $p$ . Unlike the conventional mutual information [4] where a uniform joint probability  $P(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p}) = \frac{1}{N} \sum_{i \in \mathbf{X}_p} T[(i, \hat{i}) = \{\mathbf{X}(p), \mathbf{X}(p+f_p)\}]$  is used to compute  $H^w(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p})$ . WMI employs spatially varying non-uniform joint probability  $P^w(\mathbf{X}_p, \hat{\mathbf{X}}_{p+f_p}) = \frac{1}{N} \sum_{i \in \mathbf{X}_p} w_p(f_p) T[(i, \hat{i}) = \{\mathbf{X}(p), \hat{\mathbf{X}}(p+f_p)\}]$ .  $T[\cdot]$  is logistic operator providing 1 when the argument is true. A spatially varying weight  $w_p$  is defined as follows:

$$w_p(f_p) = \frac{1}{K} \exp(-\|p - q\|^2 / \sigma_s^2) \cdot \exp(-\|I_p - I_q\|^2 / \sigma_d^2), \tag{2}$$

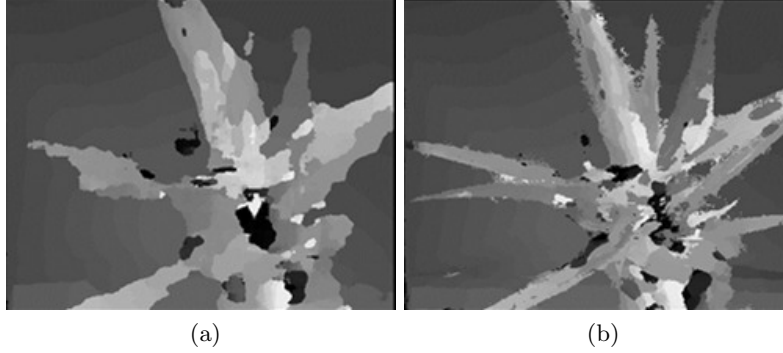


Figure 2. Comparison of initial disparities estimated with MI and WMI for the stereo pair: left image (low exposure) and right image (high exposure). (a) MI, (b) WMI.

where  $K$  is a normalization factor,  $\sigma_s$  is the parameter controlling spatial factor,  $\sigma_d$  is the parameter controlling color similarity factor,  $p$  is the center pixel in the window, and  $q$  is the neighboring pixel in the window.  $I_p$  and  $I_q$  are intensity values at  $p$  and  $q$ , respectively. Initial disparity map  $\mathbf{D}$  is estimated using winner take all (WTA) scheme with WMI such that  $\mathbf{D}(p) = \arg \max_{f_p} \mathbf{C}(p, p + f_p)$ . Fig. 2 shows the comparison of initial disparities computed using the conventional MI and the proposed WMI. As shown in Fig. 2, WMI provides more reliable results especially around an object boundary.

The reliable GCPs are computed from initial disparity map  $\mathbf{D}$  through the reliability check. The reliability of each pixel is determined by a difference between the highest cost  $\mathbf{C}'_p$  and the second highest cost  $\mathbf{C}''_p$  as depicted in Fig. 3. It is assumed that a large difference between  $\mathbf{C}'_p$  and  $\mathbf{C}''_p$  means that the highest cost value is distinct, i.e. the corresponding disparity is reliable. If cost values across different disparity candidates are similar each other, the corresponding disparity is less reliable. Based on this observation, the reliability of disparity  $\rho(p)$  at pixel  $p$  is defined as follows:

$$\rho(p) = \frac{\mathbf{C}'_p - \mathbf{C}''_p}{\mathbf{C}'_p}, \quad (3)$$

where  $\mathbf{C}'_p = \max_{f_p \in \mathbf{S}} \mathbf{C}_w(p, f_p)$  is the highest cost for each pixel in the candidate set of the disparity  $\mathbf{S}$ ,  $\mathbf{C}''_p = \max_{f_p \in \mathbf{S}'} \mathbf{C}_w(p, p + f_p)$  is the second highest cost for each pixel in the candidate set of the disparity  $\mathbf{S}' = \mathbf{S} - \{f'_p\}$  where  $f'_p$  is the best disparity.

According to the degree of reliability  $\rho(p)$ , pixels having  $\rho(p) > \tau$  are selected as ground control points where  $\tau$  is threshold constant value. Left-right consistency check [13] is also employed to eliminate half-occluded or unmatched pixels from ground control points,

## 2.2 Probabilistic Laplacian Propagation

In the conventional propagation methods, the ground control points are propagated into neighbor pixels using Laplacian propagation. However, the conventional Laplacian propagation is highly sensitive to GCPs since it assumes that they are very close to ground-truth disparity. However, the disparity of GCPs may not be a ground-truth, and thus in such a case the propagation results are possibly erroneous.

To alleviate this problem, we propose a probabilistic Laplacian propagation scheme. The conventional Laplacian propagation can be formulated in terms of energy minimization. In this framework, one seeks the labeling of  $f$  that minimizes the energy. The energy model of the conventional Laplacian propagation is given as:

$$\mathbf{E}(f) = \mathbf{E}_d(f) + \mathbf{E}_s(f), \quad (4)$$

where the data energy term  $\mathbf{E}_d(f_p)$  and the smoothness energy term  $\mathbf{E}_s(f_p)$  are defined as follows:

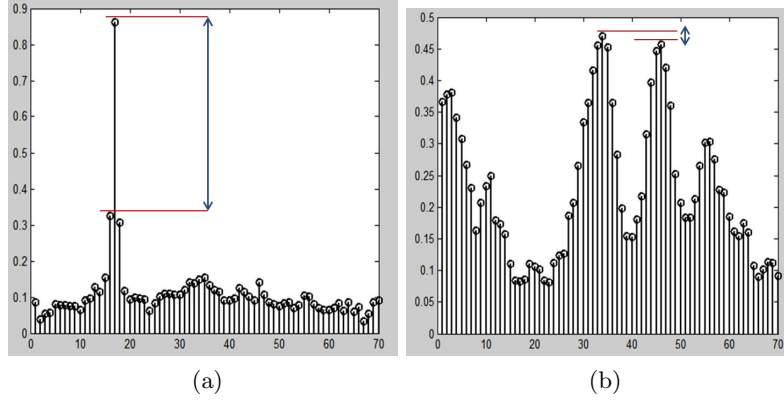


Figure 3. Comparison of reliability. (a) Case of high reliability, (b) Case of low reliability

$$\mathbf{E}_d(f) = \sum_p \mu(p)(f_p - \hat{f}_p)^2, \quad (5)$$

$$\mathbf{E}_s(f) = \sum_p \sum_{q \in \mathbf{N}(p)} \omega_{p,q} \cdot (f_p - f_q)^2, \quad (6)$$

where  $\hat{f}_p$  represents the GCPs,  $\mathbf{E}_d(f)$  encodes the penalty for the dissimilarity between the corresponding disparity and GCP disparity value with associated binary decision terms  $\mu(p)$  where  $\mu(p) = 1$  if GCP exist at pixel  $p$  and  $\mu(p) = 0$  otherwise.

$\mathbf{E}_s(f)$  imposes the constraint that two adjacent pixels should have similar disparity values according to color similarity by trying to minimize the difference between the disparity of pixel  $p$  and the weighted average of the disparities at neighbors.  $\omega_{p,q} \propto e^{-(I_p - I_q)^2 / 2\sigma_p^2}$ , which is a weighting function that sums to one, is large when intensity  $I_p$  is similar to intensity  $I_q$ .  $\sigma_p$  is the variance of the intensities in a window centered at  $p$ .

However, an inherent problem with this method is that initial false GCPs might lead to severe disparity errors during the propagation process. To lessen the influence of false GCPs, the reliability of GCPs are taken into consideration during propagation (this method is referred here as *probabilistic Laplacian propagation*). An energy model of the reliability-based probabilistic Laplacian propagation may be formulated as follows:

$$\mathbf{E}_d(f) = \sum_p \rho(p) \cdot (f_p - \hat{f}_p)^2, \quad (7)$$

where  $\mathbf{E}_d(f)$  encodes the penalty for the dissimilarity between the corresponding disparity and GCP disparity value with associated reliability term  $\rho(p)$ .

By substituting  $\rho(p)$  for  $\mu(p)$ , the reliability-based probabilistic Laplacian propagation prevent a false GCP from propagating into neighbor pixels. The energy function  $\mathbf{E}(\mathbf{F}_p)$  can be expressed in a matrix-vector form as:

$$\mathbf{E}(\mathbf{F}) = (\mathbf{F} - \hat{\mathbf{F}})^T \cdot \mathbf{P} \cdot (\mathbf{F} - \hat{\mathbf{F}}) + (\mathbf{F}^T \cdot (\mathbf{U} - \mathbf{W}) \cdot \mathbf{F}, \quad (8)$$

where the matrix  $\mathbf{P}$  is diagonal with the disparity reliability  $\mathbf{P} = \rho(p)$ ,  $\hat{\mathbf{F}}$  is vector form of the  $\hat{f}_p$ , the matrix  $\mathbf{U}$  is a diagonal matrix whose diagonal elements are the sum of the affinities  $\{\omega_{p,q}\}$  (identity matrix), and the matrix  $\mathbf{W}$  is diagonal and contains  $\omega_{p,q}$  on the row that corresponds to the interaction between the  $p$ -th and  $q$ -th elements.

The minimum of this discrete quadratic form can be obtained by setting  $\nabla \mathbf{E}(\mathbf{F}) = 0$ , which amounts to solving the following linear system as:

$$(\mathbf{P} + \mathbf{U} - \mathbf{W}) \cdot \mathbf{F} = \mathbf{P} \cdot \hat{\mathbf{F}}. \quad (9)$$

This linear system is easily solved as linear solvers.  $\mathbf{F}$  is the final disparity map.

### 3. EXPERIMENTAL RESULTS

This section evaluate the performance of the proposed method in estimating the disparity map for stereo pairs under severe radiometric variations. The proposed method is compared to complex optimization based methods with several robust cost functions, MI [4], Census [7], ANCC [3], and MDCC [4]. Graph-cut (GC) [17] is employed as a optimization algorithm, and for implementing GC we used the source code provided in [18]. Also, the performance of reliability based Laplacian propagation is compared with the conventional Laplacian propagation, in which the same GCPs are used computed by the proposed scheme for both propagation methods. We refer MI and GC based method as MI+GC, Census and GC based method as Census+GC, ANCC and GC based method ANCC+GC, the conventional Laplacian propagation with reliable GCP as LP, and the proposed propabilistic Laplacian propagation as PLP.

For the evaluation, we used Middlebury datasets including Aloe, Art, Dolls, and Moebius [19]. Each data set includes three different illuminations and camera exposures. The index of illumination ranges from 1 to 3, and the index of exposure ranges from 0 to 2. The performance of the proposed method is evaluated in terms of illumination variation robustness and exposure variation robustness. In the first experiment (illumination variation robustness), the exposure of a left image is fixed to index 1 but the illumination of a right image was changed from index 1 to index 3, and in the second experiment (exposure variation robustness) the illumination of a left image is fixed to index 1 but the exposure of a right image was changed from index 0 to index 2.

Quantitative and qualitative comparison results are given in Fig. 4 and Fig. 5, respectively. The optimization based methods provide erroneous disparities as shown in Fig. 5 since errors in an initial cost volume cannot be suppressed in optimization process. In contrast, the proposed propagation based method provide more reliable disparities because the disparity map is estimated by the propagation of reliably matched pixels. In addition, the proposed PLP shows more reliable disparity compared to the conventional LP. Note that the result of LP is also a part of the overall proposed framework. As shown in Fig. 4, the proposed method presents 14.4% and 4.6% pixel errors for illumination and exposure variations, respectively, and outperforms the compared methods. The results given in this section indicate that the proposed method based on probabilistic Laplacian propagation can be an alternative approach robust to radiometric distortions, and the performance is promising.

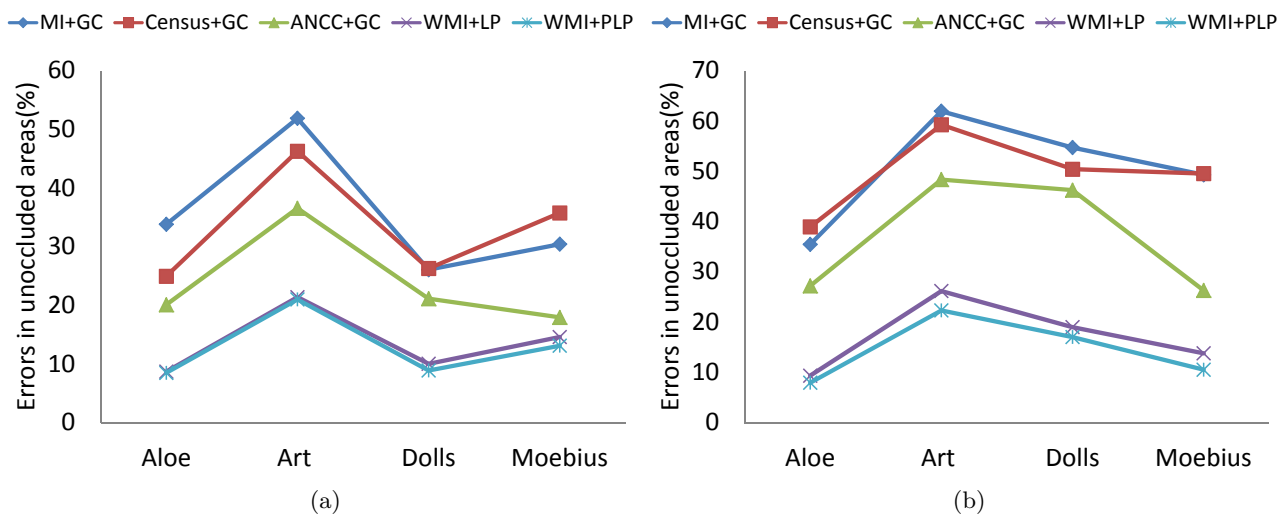


Figure 4. Performance of the proposed method. (a) Illumination variations. (b) Exposure variations.

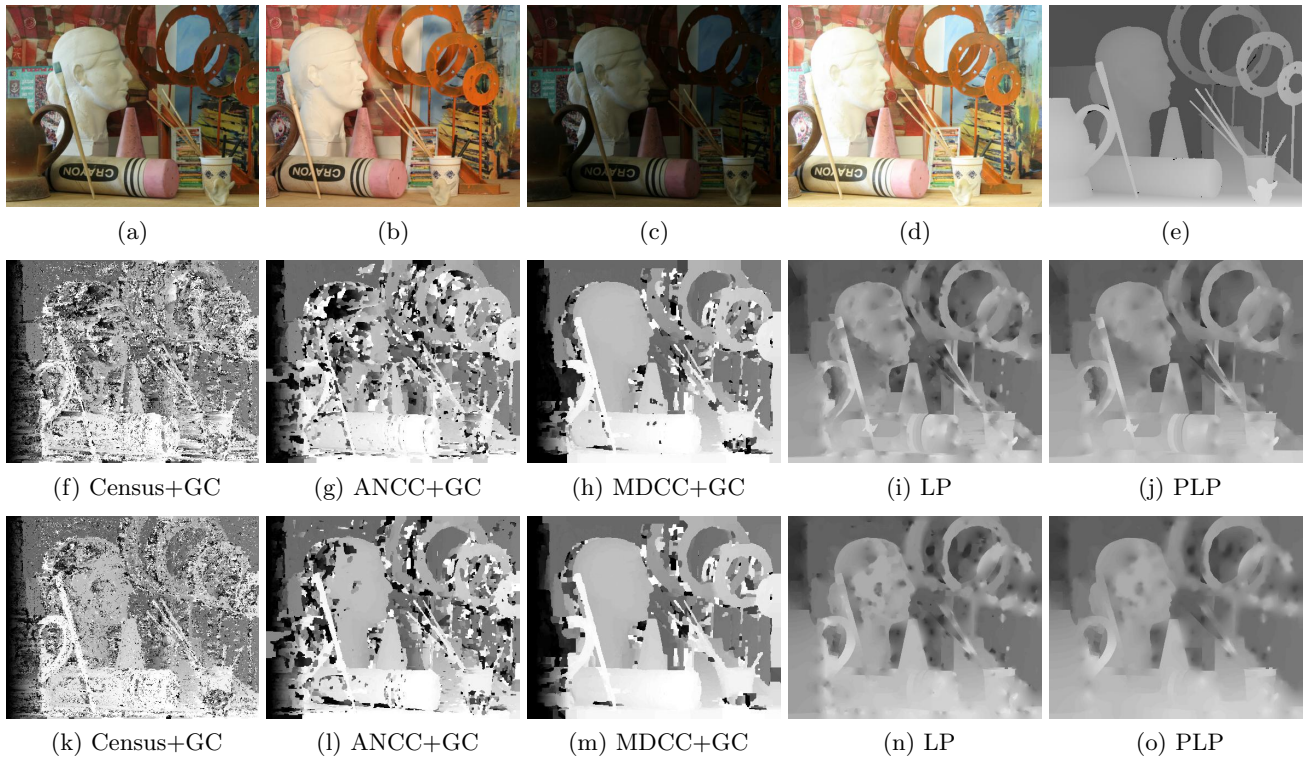


Figure 5. Qualitative comparison for Art image pair with varied illumination and exposure. (a) left image with illumination index 1 and exposure index 1, (b) right image with illumination index 3 and exposure index 1, (c) left image with illumination index 1 and exposure index 0, (d) right image with illumination index 1 and exposure index 2, (e) ground truth disparity, (f)-(j) estimated disparities for the image pair (a) and (b), (k-o) estimated disparities for the image pair (c) and (d).

#### 4. CONCLUSION

A robust stereo matching algorithm is proposed for stereo pairs under different radiometric conditions, based on WMI and reliability based probabilistic Laplacian propagation. The reliable GCPs are computed from initial disparity map estimated using WMI and reliability check process. The reliable GCPs are then propagated into neighbor pixels by the probabilistic Laplacian propagation in which the propagation of less reliable GCP is suppressed. Experimental results show that the proposed method outperforms the complex optimization based methods in radiometric variations. The proposed framework is also applicable to the other dense correspondence problems such as optical flow, scene flow, and dense feature matching.

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